Kenngrößen periodischer Funktionen:

In den Beispielen wird immer die Spannung verwendet. Alle Funktionen und ihre Kenngrößen sind auch auf den Strom anwendbar. Es gilt für alle Kenngrößen das ohmsche Gesetz!

Informieren Sie sich bitte auch im Lernprogramm "Periodische Funktionen".

Die letzte Bezeichnung ist die US-amerikanische Variante

Spitzenwert:

$$U_s = \hat{U} = V_\mathrm{p}$$

$$I_s = \frac{U_s}{R}$$

Wert Spitze-Spitze:

$$U_\mathrm{ss} = V_\mathrm{pp}$$

$$I_{ss} = \frac{U_{ss}}{R}$$

Effektivwert:

$$U = \sqrt{\frac{1}{T}\int_t^{t+T} \left(u(t')\right)^2 \mathrm{d}t'} = V_\mathrm{RMS}$$

$$I = \frac{U}{R}$$

Mittelwert:

$$\overline{U} = \frac{1}{T}\int_t^{t+T} u(t')\mathrm{d}t'$$

$$\overline{I} = \frac{\overline{U}}{R}$$

Gleichrichtwert

$$\overline{|U|} = \frac{1}{T}\int_{t}^{t+T} |u(t')|\,\mathrm{d}t'$$

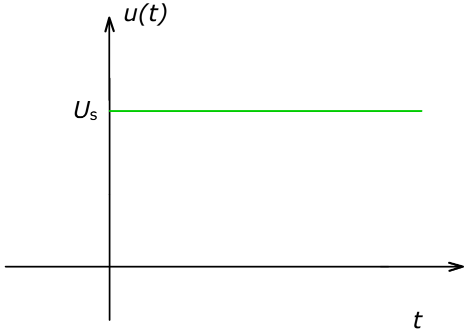
$$\overline{|I|}_s = \frac{\overline{|U|}}{R}$$

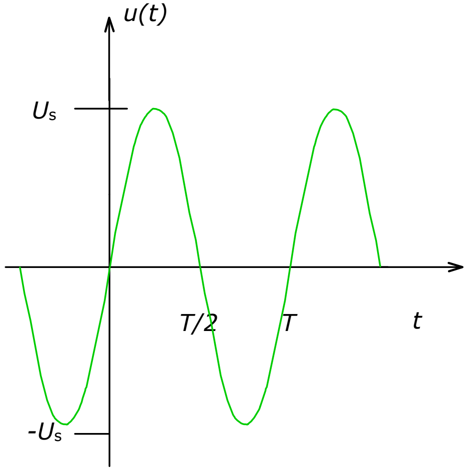
Der Mittelwert entspricht der bewerteten Fläche (Vorzeichen) unter der Kurve und kann bei einfachen Zeitverläufen (Dreieck, Rechteck) über geometrische Formeln errechnet werden!
Gleiches gilt für den Gleichrichtwert, nur wird hier der Betrag der Fläche angenommen.

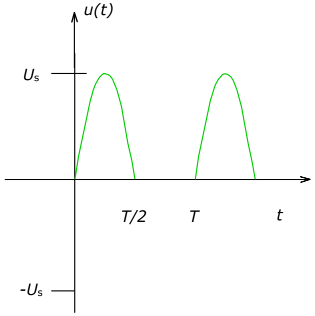
Wichtige Integrale:

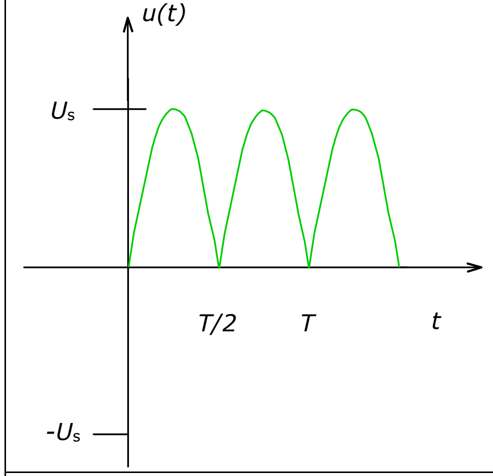
$\int_{t}^{t+T} U_s \mathrm{d}t' = U_s T$ $\int_{0}^{T} \sin\left(2\pi \frac{t}{T}\right)\mathrm{d}t = 0$ $\int_{\alpha}^{\pi} \sin(\varphi)\cdot \mathrm{d}\varphi = 1+\cos(\alpha)$ $\int_{\alpha}^{\pi} \sin^2(\varphi)\cdot \mathrm{d}\varphi = \frac{\pi-\alpha}{2}+\frac{1}{4}\sin(2\alpha)$ $\int_{\pi+\alpha}^{2\pi} \sin^2(\varphi)\cdot \mathrm{d}\varphi = \frac{\pi-\alpha}{2}+\frac{1}{4}\sin(2\alpha)$ $\int_{0}^{\alpha} \sin(\varphi)\cdot \mathrm{d}\varphi = 1-\cos(\alpha)$ $\int_{0}^{\alpha} \sin^2(\varphi)\cdot \mathrm{d}\varphi = \frac{\alpha}{2}-\frac{1}{4}\sin(2\alpha)$	$\int_{0}^{T} \sin^2\left(2\pi \frac{t}{T}\right)\mathrm{d}t = \left[\frac{t}{2}-\frac{T}{8\pi}\sin\left(4\pi\frac{t}{T}\right)\right]_0^T = \frac{T}{2}$ $\int_{0}^{T/2} \sin^2\left(2\pi \frac{t}{T}\right)\mathrm{d}t = \left[\frac{t}{2}-\frac{T}{8\pi}\sin\left(4\pi\frac{t}{T}\right)\right]_0^{T/2} = \frac{T}{4}$

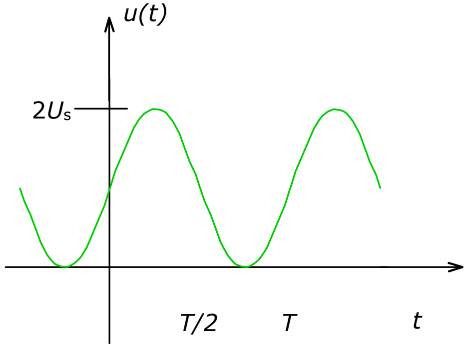
Beispiele:

<table>
<tr><td colspan="2">Konstante Spannung:

</td><td>$u(t) = U_s$</td></tr>
<tr><td>$U = \frac{1}{T}\int_0^T U_s \mathrm{d}t = U_s$
(T beliebig)</td><td>$U = \sqrt{\frac{1}{T}\int_0^T U_s^2 \mathrm{d}t} = U_s$
(T beliebig)</td><td></td></tr>
<tr><td>$U = \frac{1}{T}\int_0^T \lvert U_s \rvert \mathrm{d}t = \lvert U_s \rvert$
(T beliebig)</td><td></td><td></td></tr>
</table>

<table>
<tr>
<td colspan="2">Sinusförmige Wechselspannung:

</td>
<td>$u(t) = U_s \sin(\omega t) =$
$U_s \sin\left(2\pi \frac{t}{T}\right)$</td>
</tr>
<tr>
<td>$\overline{U} = \frac{1}{T}\int_0^T U_s \sin\left(2\pi \frac{t}{T}\right) \mathrm{d}t = 0$</td>
<td>$U = U_s \sqrt{\frac{1}{T}\int_0^T \sin^2\left(2\pi \frac{t}{T}\right) \mathrm{d}t}$
$U = \frac{U_s}{\sqrt{2}}$</td>
<td></td>
</tr>
<tr>
<td>$\left|\overline{U}_s\right| = \frac{1}{T}\int_0^T \left|U_s\right| \mathrm{d}t = \frac{2}{T} U_s \int_0^{T/2} \sin\left(2\pi \frac{\mathrm{t}}{\mathrm{T}}\right) \mathrm{d}t$
$\left|\overline{U}_s\right| = 2\frac{U_s}{\pi}$</td>
<td></td>
<td></td>
</tr>
</table>

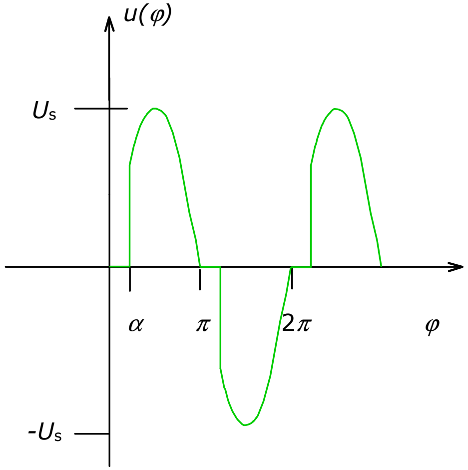
Sinusförmige Wechselspannung nach Einweggleichrichtung:  		$u(t) = U_\mathrm{s} \sin(\omega t) \qquad 0 \le t \le \frac{T}{2}$ $u(t) = 0 \qquad \frac{T}{2} < t \le T$
$\overline{U} = \frac{1}{T}\int_0^T U_\mathrm{s} \sin\left(2\pi \frac{t}{T}\right)\mathrm{d}t$ $\overline{U} = \frac{U_\mathrm{s}}{\pi}$	$U = U_s \sqrt{\frac{1}{T}\int_0^{T/2} \sin^2\left(2\pi \frac{t}{T}\right)\mathrm{d}t}$ $U = U_s \sqrt{\frac{1}{T}\frac{T}{4}}$ $U = \frac{U_s}{2}$	
$\overline{\lvert U \rvert} = \frac{1}{T}\int_0^{T/2} U_\mathrm{s} \sin\left(2\pi \frac{t}{T}\right)\mathrm{d}t$ $\overline{\lvert U \rvert} = \frac{U_\mathrm{s}}{\pi}$		

Sinusförmige Wechselspannung nach Zweiweggleichrichtung: 		Gegenüber dem Originalsinus halbiert sich die Periode! $u(t) = U_s \sin(\omega t) \qquad 0 \le t \le \frac{T}{2}$
$\overline{U} = \frac{1}{T/2}\int_0^{T/2} U_s \sin\left(2\pi\frac{t}{T}\right)\mathrm{d}t$ $\overline{U} = \frac{2U_s}{\pi}$	$U = U_s\sqrt{\frac{1}{T/2}\int_0^{T/2}\sin^2\left(2\pi\frac{t}{T}\right)\mathrm{d}t}$ $U = \frac{U_s}{\sqrt{2}}$	
$\overline{U} = \frac{1}{T/2}\int_0^{T/2} U_s \sin\left(2\pi\frac{t}{T}\right)\mathrm{d}t$ $\overline{U} = \frac{2U_s}{\pi}$		

Sinusförmige Wechselspannung mit Gleichanteil: 		$u(t)=U_s\left[1+\sin(\omega t)\right]$ $u(t)=U_s\left[1+\sin\left(2\pi\frac{t}{T}\right)\right]$
$\overline{U}=\frac{1}{T}\int_0^T U_s\left[1+\sin\left(2\pi\frac{t}{T}\right)\right]\mathrm{d}t$ $\overline{U}=U_s$	$U=U_s\sqrt{\frac{1}{T}\int_0^T\left[1+\sin\left(2\pi\frac{t}{T}\right)\right]^2\mathrm{d}t}$ $U=U_s\sqrt{\frac{1}{T}\int_0^T\left(1+2\sin\left(2\pi\frac{t}{T}\right)+\sin^2\left(2\pi\frac{t}{T}\right)\right)\mathrm{d}t}$ $U=U_s\sqrt{\left[\frac{1}{T}\left(t-\frac{2T}{2\pi}\cos\left(2\pi\frac{t}{T}\right)+\frac{t}{2}-\frac{T}{8\pi}\sin\left(4\pi\frac{t}{T}\right)\right)\right]_0^T}$ $U=U_s\sqrt{\frac{3}{2}}$	
$\left\lvert\overline{U}\right\rvert=\frac{1}{T}\int_0^T U_s\left[1+\sin\left(2\pi\frac{t}{T}\right)\right]\mathrm{d}t$ $\left\lvert\overline{U}\right\rvert=U_s$		

<table>
<tr>
<td colspan="2">

Sinus mit Phasenanschnitt:



</td>
<td>

Da der Phasenanschnitt meist in Grad angegeben wird, wird hier die Spannung über dem Winkel dargestellt:

$u(t) = 0 \qquad 0 \le \varphi \le \alpha$

$u(t) = U_s \sin\alpha \qquad \alpha < \varphi \le \pi$

$u(t) = 0 \qquad \pi < \varphi \le \pi + \alpha$

$u(t) = U_s \sin\alpha \qquad \pi + \alpha < \varphi \le 2\pi$

</td>
</tr>
<tr>
<td>

$\overline{U} = \frac{U_s}{T}\left(\int_{\alpha}^{\pi} \sin(\varphi)\,\mathrm{d}\varphi + \int_{\pi+\alpha}^{2\pi} \sin(\varphi)\,\mathrm{d}\varphi\right)$

$\overline{U} = 0$

</td>
<td>

$U = \sqrt{\frac{U_s^2}{2\pi}\left(\int_{\alpha}^{\pi} \sin^2(\varphi)\,\mathrm{d}\varphi + \int_{\pi+\alpha}^{2\pi} \sin^2(\varphi)\,\mathrm{d}\varphi\right)}$

$U = U_s\sqrt{\left(\frac{1}{2\pi}\left(\pi - \alpha + \frac{1}{2}\sin(2\alpha)\right)\right)}$

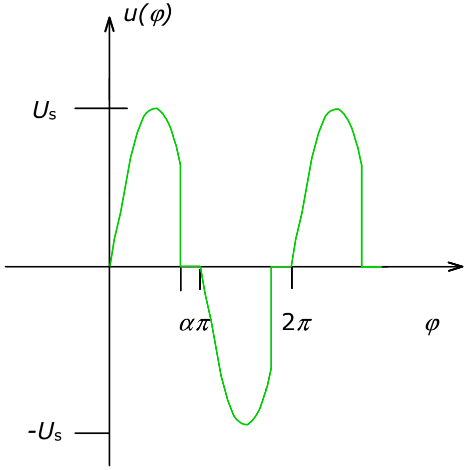
</td>
<td></td>
</tr>
<tr>
<td>

$\overline{U} = \frac{U_s}{2\pi}\left(\int_{\alpha}^{\pi} \sin(\varphi)\,\mathrm{d}\varphi - \int_{\pi+\alpha}^{2\pi} \sin(\varphi)\,\mathrm{d}\varphi\right)$

$\overline{U} = \frac{U_s}{2\pi} 2\left(\int_{\alpha}^{\pi} \sin(\varphi)\,\mathrm{d}\varphi\right)$

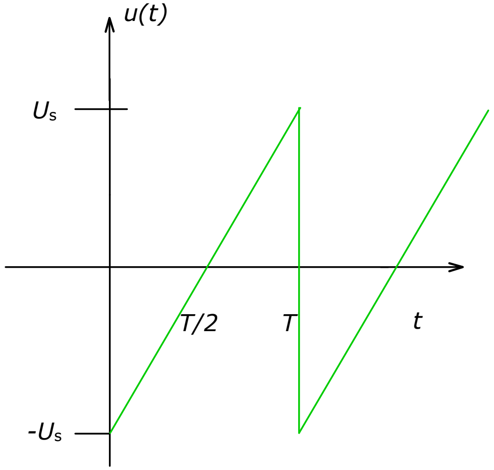
$\overline{U} = \frac{2U_s}{2\pi}(1 + \cos\alpha) = \frac{U_s}{\pi}(1 + \cos\alpha)$

</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</table>

Sinus mit Phasenabschnitt:  		Da der Phasenabschnitt meist in Grad angegeben wird, wird hier die Spannung über dem Winkel dargestellt: $u(t) = U_s \sin\alpha \quad 0 \le \varphi \le \alpha$ $u(t) = 0 \quad \alpha < \varphi \le \pi$ $u(t) = U_s \sin\alpha \quad \pi < \varphi \le \pi + \alpha$ $u(t) = 0 \quad \pi + \alpha < \varphi \le 2\pi$
$\overline{U} = \frac{U_s}{T}\left(\int_0^{\alpha} \sin(\varphi)\,d\varphi + \int_{\pi}^{\pi+\alpha} \sin(\varphi)\,d\varphi\right)$ $\overline{U} = 0$	$U = \sqrt{\frac{U_s^2}{2\pi}\left(\int_0^{\alpha} \sin^2(\varphi)\,d\varphi + \int_{\pi}^{\pi+\alpha} \sin^2(\varphi)\,d\varphi\right)}$ $U = U_s\sqrt{\left(\frac{1}{2\pi}\left(\alpha - \frac{1}{2}\sin(2\alpha)\right)\right)}$	
$\overline{U} = \frac{U_s}{2\pi}\left(\int_{\alpha}^{\pi} \sin(\varphi)\,d\varphi - \int_{\pi+\alpha}^{2\pi} \sin(\varphi)\,d\varphi\right)$ $\overline{U} = \frac{U_s}{2\pi}2\left(\int_{\alpha}^{\pi} \sin(\varphi)\,d\varphi\right)$ $\overline{U} = \frac{2U_s}{2\pi}(1+\cos\alpha) = \frac{U_s}{\pi}(1+\cos\alpha)$		

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="2">

Sägezahnförmige Wechselspannung:



</td><td>

$u(t) = U_{\mathrm{s}} \left(\frac{2t}{T} - 1 \right) \qquad 0 \le t \le T$

</td></tr>
<tr><td>

$\overline{U} = \frac{1}{T} \int_0^T U_{\mathrm{s}} \left(\frac{2t}{T} - 1 \right) \mathrm{d}t$

$\overline{U} = U_{\mathrm{s}} \frac{1}{T} \left[\frac{t^2}{T} - t \right]_0^T$

$\overline{U} = U_{\mathrm{s}} \frac{1}{T} \left[\frac{t^2}{T} - t \right]_0^T = 0$

</td><td>

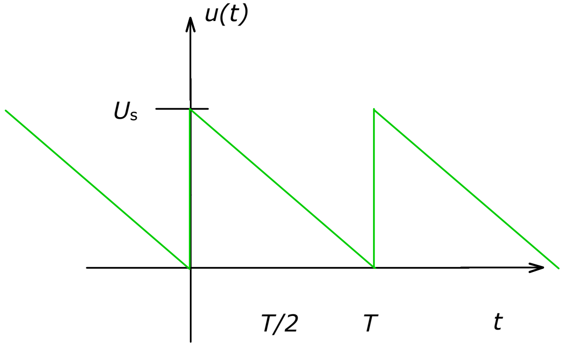
$U = U_{\mathrm{s}} \sqrt{\frac{1}{T} \int_0^T \left(\frac{2t}{T} - 1 \right)^2 \mathrm{d}t}$

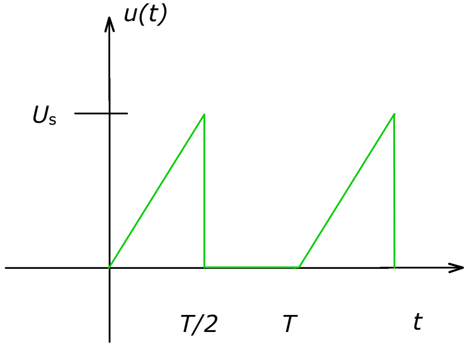
$U = U_{\mathrm{s}} \sqrt{\frac{1}{T} \int_0^T \left(\frac{4t^2}{T^2} - \frac{4t}{T} + 1 \right) \mathrm{d}t}$

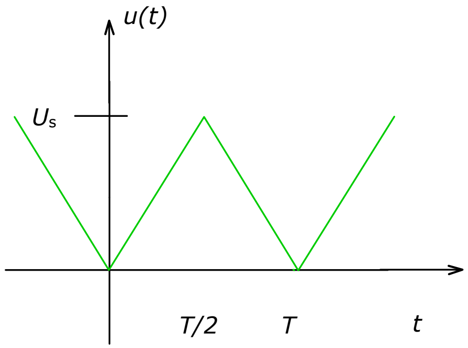
$U = U_{\mathrm{s}} \sqrt{\frac{1}{T} \left[\frac{4t^3}{3T^2} - \frac{2t^2}{T} + t \right]_0^T}$

$U = \frac{U_{\mathrm{s}}}{\sqrt{3}}$

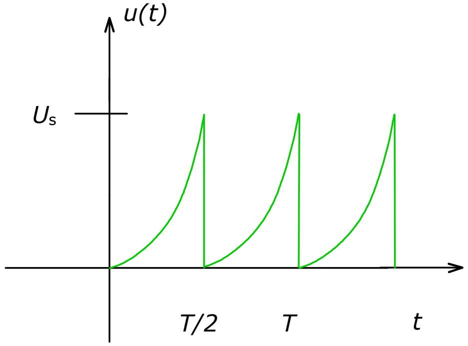
</td><td></td></tr>
</table>

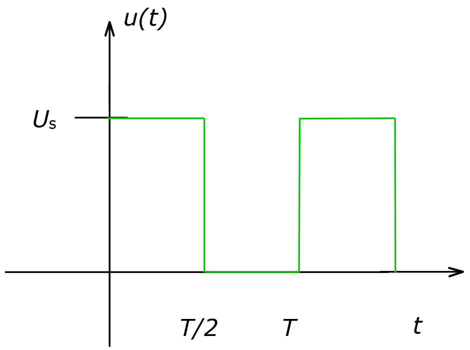
$\left\|\overline{U}\right\|=\frac{U_s}{T}\int_0^{T/2}\left(\frac{2t}{T}-1\right)\mathrm{d}t+\int_{T/2}^{T}\left(1-\frac{2t}{T}\right)\mathrm{d}t$ $\left\|\overline{U}\right\|=\frac{U_s}{T}\left(\frac{T}{4}+\frac{T}{4}\right)=\frac{U_s}{2}$		
Sägezahnförmige Wechselspannung:  		$u(t)=U_s\left(1-\frac{t}{T}\right)$
$\overline{U}=U_s\frac{1}{T}\int_0^T\left(1-\frac{t}{T}\right)\mathrm{d}t$ $\overline{U}=U_s\frac{1}{T}\left[t-\frac{t^2}{2T}\right]_0^T$ $\overline{U}=\frac{U_s}{2}$ Über die Fläche unter der Kurve: $\overline{U}=\frac{1}{T}\frac{U_sT}{2}=\frac{U_s}{2}$	$U=U_s\sqrt{\frac{1}{T}\int_0^T\left(1-\frac{t}{T}\right)^2\mathrm{d}t}$ $U=U_s\sqrt{\frac{1}{T}\int_0^T\left(1-2\frac{t}{T}+\frac{t^2}{T^2}\right)\mathrm{d}t}$ $U=U_s\sqrt{\frac{1}{T}\left[t-\frac{t^2}{T}+\frac{t^3}{3T^2}\right]_0^T}$ $U=\frac{U_s}{\sqrt{3}}$	

<table>
<tr>
<td>$|\overline{U}| = U_s \frac{1}{T}\int_0^T \left(1-\frac{t}{T}\right)\mathrm{d}t$
$|\overline{U}| = U_s \frac{1}{T}\left[t-\frac{t^2}{2T}\right]_0^T$
$|\overline{U}| = \frac{U_s}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td colspan="2">Sägezahnförmige Wechselspannung:

</td>
<td>$u(t) = U_s \frac{2t}{T} \qquad 0 \le t \le \frac{T}{2}$
$u(t) = 0 \qquad \frac{T}{2} < t \le T$</td>
</tr>
<tr>
<td>$\overline{U} = U_s \frac{1}{T}\int_0^{T/2} \frac{2t}{T}\mathrm{d}t$
$\overline{U} = U_s \frac{T}{T}\left[\left(\frac{t}{T}\right)^2\right]_0^{T/2}$
$\overline{U} = \frac{U_s}{4}$
Über die Fläche:
$U = \frac{1}{T}\frac{U_s\frac{T}{2}}{2} = \frac{U_s}{4}$</td>
<td>$U = U_s\sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2t}{T}\right)^2\mathrm{d}t}$
$U = U_s\sqrt{\frac{1}{T}\int_0^{T/2}\frac{4t^2}{T^2}\mathrm{d}t}$
$U = U_s\sqrt{\frac{1}{T}\left[\frac{4t^3}{3T^2}\right]_0^{T/2}}$
$U = \frac{U_s}{\sqrt{6}}$</td>
<td></td>
</tr>
</table>

$\left\|\overline{U}\right\| = U_s \frac{1}{T}\int_0^{T/2}\frac{2t}{T}\,\mathrm{d}t$ $\left\|\overline{U}\right\| = U_s \frac{T}{T}\left[\left(\frac{t}{T}\right)^2\right]_0^{T/2}$ $\left\|\overline{U}\right\| = \frac{U_s}{4}$		
Dreieckförmige Wechselspannung: 		$u(t) = U_s\left(1-\frac{2t}{T}\right) \qquad 0 \le t \le \frac{T}{2}$ $u(t) = U_s\left(\frac{2\left(t-\frac{T}{2}\right)}{T}\right) \qquad \frac{T}{2} < t \le T$

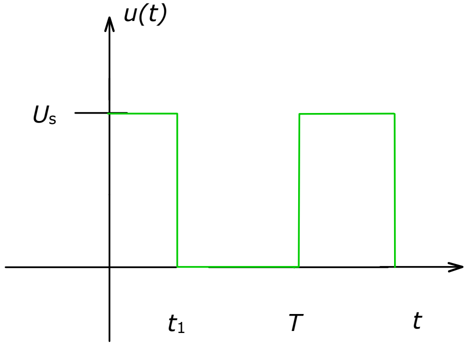
$\overline{U} = U_s \frac{1}{T}\left[\int_0^{T/2}\left(1-\frac{2t}{T}\right)\mathrm{d}t + \int_{T/2}^{T}\left(\frac{2\left(t-\frac{T}{2}\right)}{T}\right)\mathrm{d}t\right]$ $\overline{U} = U_s \frac{1}{T}\left\{\left[\left(t-\frac{t^2}{T}\right)\right]_0^{T/2} + \left[\frac{\left(t-\frac{T}{2}\right)^2}{T}\right]_{T/2}^{T}\right\}$ $\overline{U} = \frac{U_s}{2}$	$U = U_s\sqrt{\frac{1}{T}\int_0^{T/2}\left(1-\frac{2t}{T}\right)^2 \mathrm{d}t + \frac{1}{T}\int_{T/2}^{T}\left(\frac{2\left(t-\frac{T}{2}\right)}{T}\right)^2 \mathrm{d}t}$ $U = U_s\sqrt{\left[\frac{1}{T}\frac{-T}{2}\frac{1}{3}\left(1-\frac{2t}{T}\right)^3\right]_0^{T/2} + \left[\frac{1}{T}\frac{T}{2}\frac{1}{3}\left(\frac{2\left(t-\frac{T}{2}\right)}{T}\right)^3\right]_{T/2}^{T}}$ $U = \frac{U_s}{\sqrt{3}}$	
$\left\|\overline{U}\right\| = U_s \frac{1}{T}\left[\int_0^{T/2}\left(1-\frac{2t}{T}\right)\mathrm{d}t + \int_{T/2}^{T}\left(\frac{2\left(t-\frac{T}{2}\right)}{T}\right)\mathrm{d}t\right]$ $\left\|\overline{U}\right\| = U_s \frac{1}{T}\left\{\left[\left(t-\frac{t^2}{T}\right)\right]_0^{T/2} + \left[\frac{\left(t-\frac{T}{2}\right)^2}{T}\right]_{T/2}^{T}\right\}$ $\left\|\overline{U}\right\| = \frac{U_s}{2}$		

Parabeläste: 		$u(t) = U_s \left(\frac{t}{T}\right)^2$
$\overline{U} = U_s \frac{1}{T} \int_0^T \left(\frac{t}{T}\right)^2 \mathrm{d}t$ $\overline{U} = U_s \frac{T}{3T} \left[\left(\frac{t}{T}\right)^3\right]_0^T$ $\overline{U} = \frac{U_s}{3}$	$U = U_s \sqrt{\frac{1}{T} \int_0^T \left(\frac{t}{T}\right)^4 \mathrm{d}t}$ $U = U_s \sqrt{\frac{T}{T} \left[\frac{1}{5}\left(\frac{t}{T}\right)^5\right]_0^T}$ $U = \frac{U_s}{\sqrt{5}}$	
$\lvert\overline{U}\rvert = U_s \frac{1}{T} \int_0^T \left(\frac{t}{T}\right)^2 \mathrm{d}t$ $\lvert\overline{U}\rvert = U_s \frac{T}{3T} \left[\left(\frac{t}{T}\right)^3\right]_0^T$ $\lvert\overline{U}\rvert = \frac{U_s}{3}$		

Rechteckförmige Wechselspannung:  		$u(t) = U_s \qquad 0 \le t \le \frac{T}{2}$ $u(t) = 0 \qquad \frac{T}{2} < t \le T$
$\overline{U} = \frac{U_s}{T} \int_0^{T/2} \mathrm{d}t$ $\overline{U} = \frac{U_s}{2}$	$U = \sqrt{\frac{U_s^2}{T} \int_0^{T/2} \mathrm{d}t}$ $U = \frac{U_s}{\sqrt{2}}$	
$\lvert\overline{U}\rvert = \frac{U_s}{T} \int_0^{T/2} \mathrm{d}t$ $\lvert\overline{U}\rvert = \frac{U_s}{2}$		

<table>
<tr><td colspan="2">

Rechteckförmige Wechselspannung:



</td><td>

$u(t) = U_s \quad 0 \le t \le t_1$

$u(t) = 0 \quad t_1 < t \le T$

Das Verhältnis $\frac{t_1}{T}$ wird Tastgrad genannt.

</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>

$\overline{U} = \frac{U_s}{T}\int_0^{t_1} \mathrm{d}t$

$\overline{U} = U_s \frac{t_1}{T}$

</td><td>

$U = \sqrt{\frac{U_s^2}{T}\int_0^{T/2} \mathrm{d}t}$

$U = U_s\sqrt{\frac{t_1}{T}}$

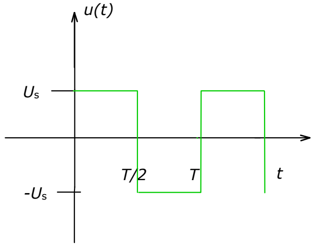
</td><td></td></tr>
<tr><td>

$\left|\overline{U}\right| = \frac{U_s}{T}\int_0^{t_1} \mathrm{d}t$

$\left|\overline{U}\right| = U_s \frac{t_1}{T}$

</td><td></td><td></td></tr>
<tr><td colspan="2">

Rechteckförmige Wechselspannung:

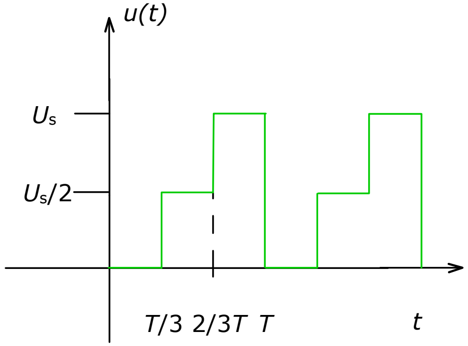


</td><td>

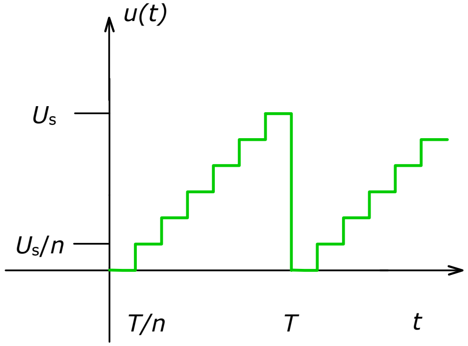
$u(t) = U_s \quad 0 \le t \le \frac{T}{2}$

$u(t) = -U_s \quad \frac{T}{2} < t \le T$

</td></tr>
</table>

$\overline{U} = \frac{U_s}{T}\left(\int_0^{T/2} dt - \int_{T/2}^{T} dt\right)$ $\overline{U} = 0$	$U = \sqrt{\frac{U_s^2}{T}\left(\int_0^{T/2} dt + \int_{T/2}^{T} dt\right)}$ $U = U_s$	
$\overline{U} = \frac{U_s}{T}\left(\int_0^{T/2} dt + \int_{T/2}^{T} dt\right)$ $\overline{U} = U_s$		
Treppenförmige Wechselspannung  		$u(t) = 0 \quad 0 \le t \le \frac{T}{3}$ $u(t) = \frac{U_s}{2} \quad \frac{T}{3} < t \le \frac{2T}{3}$ $u(t) = U_s \quad \frac{2T}{3} < t \le T$
$\overline{U} = \frac{U_s}{T}\left(\int_{T/3}^{2T/3} \frac{1}{2} dt + \int_{2T/3}^{T} 1 dt\right)$ $\overline{U} = \frac{U_s}{2}$	$U = \sqrt{\frac{U_s^2}{T}\left(\int_{T/3}^{2T/3} \left(\frac{1}{2}\right)^2 dt + \int_{2T/3}^{T} dt\right)}$ $U = U_s\sqrt{\frac{5}{12}}$	
$\lvert\overline{U}\rvert = \frac{U_s}{T}\left(\int_{T/3}^{2T/3} \frac{1}{2} dt + \int_{2T/3}^{T} 1 dt\right)$ $\lvert\overline{U}\rvert = \frac{U_s}{2}$		

Treppenförmige Wechselspannung mit n Stufen:



Bei n Stufen hat jede die Breite $\frac{T}{n}$, damit

ergibt sich für den Mittelwert:

$$\overline{U} = U_s \frac{1}{T}\left(\frac{1}{(n-1)}\frac{T}{n} + \frac{2}{(n-1)}\frac{T}{n} + \frac{3}{(n-1)}\frac{T}{n}....\right)$$

$$\overline{U} = U_s \frac{1}{n(n-1)}\left(\sum_{k=1}^{n}(k-1)\right)$$

$$= U_s \frac{1}{n(n-1)}\left(\frac{n}{2}(n-1)\right)$$

$$\overline{U} = \frac{U_s}{2}$$

Probe, für
$n \rightarrow \infty$

$$\overline{U} = \frac{U_s}{2}$$

Entsprechend Sägezahn!

$$U = U_s\sqrt{\frac{1}{T}\left(\left(\frac{1}{n-1}\right)^2\frac{T}{n} + \left(\frac{2}{n-1}\right)^2\frac{T}{n} + \left(\frac{3}{n-1}\right)^2\frac{T}{n}....\right)}$$

$$U = U_s\sqrt{\frac{1}{n(n-1)^2}\left(\sum_{k=1}^{n}(k-1)^2\right)} = U_s\sqrt{\frac{1}{n(n-1)^2}\left(\frac{n(n-1)(2n-1)}{6}\right)}$$

$$U = U_s\sqrt{\frac{1}{(n-1)}\left(\frac{(2n-1)}{6}\right)}$$

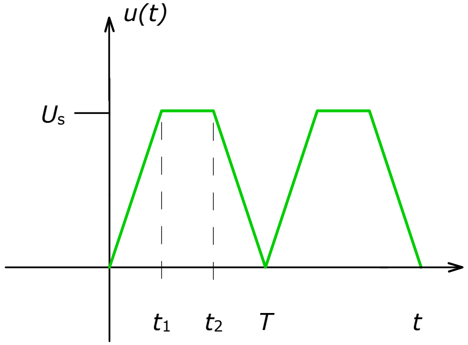
Probe, für
$n \rightarrow \infty$

$$\overline{U} = \frac{U_s}{\sqrt{3}}$$

Entsprechend Sägezahn!

Gleichrichtwert, siehe Mittelwert

Trapezförmige Wechselspannung:



$$u(t) = U_s \frac{t}{t_1} \qquad 0 \le t \le t_1$$

$$u(t) = U_s \qquad t_1 < t \le t_2$$

$$u(t) = U_s \left(1 - \frac{t - t_2}{T - t_2} \right) \qquad t_1 < t \le t_2$$

$\overline{U} = \frac{U_s}{T}\left(\int_0^{t_1}\frac{t}{t_1}\mathrm{d}t + \int_{t_1}^{t_2}\mathrm{d}t + \int_{t_2}^{T}\left(1-\frac{t-t_2}{T-t_2}\right)\mathrm{d}t\right)$ $\overline{U} = \frac{U_s}{T}\left(\frac{t_1^2}{2t_1} + (t_2-t_1) + \left[t - \frac{T-t_2}{2}\left(\frac{t-t_2}{T-t_2}\right)^2\right]_{t_2}^{T}\right)$ $\overline{U} = \frac{U_s}{T}\left(\frac{t_1}{2} + (t_2-t_1) + \left(T - t_2 - \frac{T-t_2}{2}\right)\right)$ $\overline{U} = \frac{U_s}{T}\left(\frac{t_1}{2} + (t_2-t_1) + \frac{T-t_2}{2}\right)$ Probe: $t_1 = 0 \quad t_2 = T$ (Gleichspannung) $\overline{U} = U_s$ $t_1 = t_2 = \frac{T}{2}$ (Dreieck) $\overline{U} = \frac{U_s}{2}$	$U = U_s\sqrt{\frac{1}{T}\left(\int_0^{t_1}\left(\frac{t}{t_1}\right)^2\mathrm{d}t + \int_{t_1}^{t_2}\mathrm{d}t + \int_{t_2}^{T}\left(1-\frac{t-t_2}{T-t_2}\right)^2\mathrm{d}t\right)}$ $U = U_s\sqrt{\frac{1}{T}\left(\left[\frac{t_1}{3}\left(\frac{t}{t_1}\right)^3\right]_0^{t_1} + (t_2-t_1) + \left[-\frac{T-t_2}{3}\left(1-\frac{t-t_2}{T-t_2}\right)^3\right]_{t_2}^{T}\right)}$ $U = U_s\sqrt{\frac{1}{T}\left(\frac{t_1}{3} + (t_2-t_1) + \frac{T-t_2}{3}\right)}$ $U = U_s\sqrt{\frac{1}{3T}\left(T + 2(t_2-t_1)\right)}$ Probe: $t_1 = 0 \quad t_2 = T$ (Gleichspannung) $U = U_s\sqrt{\frac{3T}{3T}}$ $t_1 = t_2 = \frac{T}{2}$ (Dreieck) $\overline{U} = U_s\sqrt{\frac{T}{3T}} = U_s\sqrt{\frac{1}{3}}$	
Gleichrichtwert, siehe Mittelwert!		